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ALTERNATING-MOVE PREPLAYS AND *vN - M* STABLE SETS IN TWO PERSON STRATEGIC FORM GAMES

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Alternating-Move Preplays and $vN - M$ Stable Sets in Two Person Strategic Form Games*

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Abstract

The alternating-move preplay procedure proposed by Bhaskar [1989] is studied in detail. It is shown that the preplay procedure works well in typical 2×2 two person games like the prisoner's dilemma, a pure coordination game, as well as in a price-setting duopoly game. A sort of the Folk Theorem is, however, shown to hold in the prisoner's dilemma with continuous strategy spaces. In addition, relations are studied between $vN - M$ stable sets in strategic form games and equilibria in their extensions with preplays.

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1 Introduction

An alternating-move preplay negotiation procedure for two-person games was proposed by Bhaskar [1989] in the context of a price-setting duopoly. The preplay proceeds as follows. One of the players, say player 1, first announces the price that he intends to take; and then player 2 announces his price. Player 1 is now given the opion of changing his price. If he does so, player 2 can change his price. The process continues in this manner; and it comes to an end when one of the two players chooses not to change his price. Bhaskar succeeded in showing that through this process only the monopoly price pair can be attained in equilibrium where the equilibrium is the subgame perfect equilibrium with undominated strategies.

One of the aims of this paper is to examine the validity of the alternating-move preplay process in other two-person games. In addition to the conditions that Bhaskar imposed on equilibria, we require that strategies in equilibrium be Markov (or stationary). It will be shown that the preplay process works well in typical 2×2 games such as the prisoner's dilemma and a pure coordination game. The pair of (Cooperation, Cooperation) and a Pareto optimal strategy pair are obtained as the unique equilibrium outcome, respectively. Further in the price-setting duopoly it will be shown that the monopoly price pair can be reached even if the preplay starts from any price pair. The preplay, however, does not always work well. In fact, a sort of the Folk Theorem is shown to hold in the prisoner's dilemma with continous strategy spaces: in the game every individual rational outcome can be attained as an equilibrium outcome.

Another objective of this paper is to study the von Neumann and Morgenstern ($vN - M$) stable sets in two-person strategic form games. Recently Greenberg [1990] proposed a way to apply $vN - M$ stable sets, or at least its spirit, to strategic form games by appropriately introducing a dominance relation on the space of strategy combinations. Later studies, Chwe [1992] and Muto and Okada [1992], however, revealed that a modification of the dominance relation is desirable as Harsanyi [1974] already pointed out in his study of the $vM - N$ stable set in characteristic function form games. Following Harsanyi's discussion, we will study relations between $vN - M$ stable sets in strategic form games and equilibria in their extended games with preplays.

The remainder of the paper is organized as follows. In Section 2, the alternating-move preplay process is described; and its rigorous formulation as an extensive form game is presented in Section 3 together with the definition of the equilibrium concept which we use throughout the paper. Section 4 studies equilibria in typical 2×2 games, the prisoner's dilemma, a pure coordination game and the battle of the sexes. The price-setting duopoly game is studied in detail in Section 5. Section 6 presents a sort of the Folk Theorem in a continuous version of the prisoner's dilemma. Section 7 studies relations between $vN-M$ stable sets and equilibria in the extended game with preplays. The paper ends in Section 8 with short remarks.

2 The Extended Game with Alternating-Move Preplays

Throughout the paper we will work on the following two-person game:

$$G=(N=\{1,2\},\{X_i\}_{i=1,2},\{u_i\}_{i=1,2})$$

where $N=\{1,2\}$ is the set of players, X_i, $i=1,2$, is player i's action set and u_i, $i=1,2$, is player i's payoff function, i.e., a real valued function on $X=X_1\times X_2$. We assume u_i takes nonnegative values.

The alternating-move preplays, proposed by Bhaskar [1989], proceed as follows. One of the players, say player 1, moves first and announces the action $x_1\in X_1$ that he intends to take. The first player to move is determined in advance of the preplays.[1] Then player 2 announces an action $x_2\in X_2$. Player 1 now has the option of changing his action to x_1'. If he does so, player 2 can change his action to x_2' and so on. The preplay process comes to an end when any of the two players chooses not to change.

Let $x^k=(x_1^k,x_2^k)$ be the action combination at the end of the kth period. For convenience let $x^1=x_1^1$: x_1^1 is the action that player 1 announces at the 1st period. Suppose the preplay process ends at the Kth period with player i's turn; thus $x^{K-1}=x^K$. Then since player i chooses not to move, he is satisfied with his action $x_i^K=x_i^{K-2}$ against j's action $x_j^K=x_j^{K-1}$. Further player j's action x_j^{K-1} is his response to player i's x_i^{K-2}. Thus both players are satisfied with the action combination x^K. Player i will be paid $u_i(x^K)$, $i=1,2$. If the equality $x^K=x^{K-1}$ never arises, then the game will go on indefinitely. In this event, we define the players' payoffs are zero.[2]

Hereafter, we will call this alternating-move game the extended game of G.

[1] If the game is symmetric with respect to players, such as the price-setting duopoly game that Bhaskar [1989] studied, it does not matter who moves first. In asymmetric games, of course, final outcomes are dependent upon who moves first. See the battle of the sexes in Section 3.

[2] Following Bhaskar [1989] and Harsanyi [1974], we assume this rather artificial condition on players' payoffs. See Section 8 concerning this assumption.

3 Formal Description of the Extended Game

In the following we describe the extended game in which player 1 moves first. Thus in the following player 1 (player 2, resp.) moves in odd (even, resp.) number of periods. The game in which player 2 moves first is described in the same manner.

3.1 Strategies and Payoffs

Take the kth period, and suppose actions announced up to the $(k-1)$st period are $x_1^1, x_2^2, x_1^3, \ldots, x_i^{k-1}$ where i is the player who moved at the $(k-1)$st period. Then the action combination x^l at the end of the lth period is given by

$$x^l = \begin{cases} (x_2^{l-1}, x_1^l) & \text{if } l \text{ is odd and } l \geq 3 \\ (x_1^{l-1}, x_2^l) & \text{if } l \text{ is even.} \end{cases}$$

The history up to the $(k-1)$st period is written as $h^{k-1} = (x^1, x^2, \ldots, x^{k-1})$. Let the set of all possible h^k by H^k, and let $H = \bigcup_{k=0}^{\infty} H^k$ where $H^0 = \{e\}$ and e denotes the empty history. Players' strategies, denoted by σ_1 for player 1 and σ_2 for player 2, are maps such that

$$\sigma_1 : \bigcup_{k=0}^{\infty} H^{2k} \to X_1$$

and

$$\sigma_2 : \bigcup_{k=0}^{\infty} H^{2k+1} \to X_2.$$

A strategy combination (σ_1, σ_2) is denoted by σ. The set of all starategies of player 1 (player 2, resp.) is denoted by $\Sigma_1(\Sigma_2$, resp.). The outcome (action combination) path induced by a strategy combination σ is denoted by $\pi(\sigma)$.

Player i's payoff under a strategy combination σ is given by

$$f_i(\sigma) = \begin{cases} u_i(z) & \text{if } \pi(\sigma) \text{ is of finite length, i.e., if the game} \\ & \text{ends after a finite number of periods:} \\ & z \text{ is the final outcome, i.e., } z = x^K \text{ when the game ends at the } Kth \text{ period} \\ 0 & \text{otherwise.} \end{cases}$$

3.2 Subgames

The extended game is a game with perfect information; and thus games starting from each move of players are subgames. Let h be a history up to the $(k-1)$st period, and denote by $\Gamma(h)$ the subgame starting from the kth period after the history h. Let $\sigma_i(h), i = 1,2$, be player i's strategy in $\Gamma(h)$, and let $\sigma(h) = (\sigma_1(h), \sigma_2(h))$. Denote by $\pi(\sigma(h))$ the outcome path in $\Gamma(h)$ induced by $\sigma(h)$. Player i's payoffs in $\Gamma(h)$ under $\sigma(h)$ are given by

$$f_i^h(\sigma(h)) = \begin{cases} u_i(z) & \text{if } \pi(\sigma(h)) \text{ is of finite length:} \\ & z \text{ is the final outcome in the path } \pi(\sigma(h)) \\ 0 & \text{otherwise.} \end{cases}$$

3.3 Equilibrium

Similarly to Bhaskar [1989], we require equilibrium strategies to be subgame perfect and also require that in equilibria the strategies played after any history should not be weakly dominated. The latter is defined in the following manner. Take a subgame $\Gamma(h)$, and take player i's two strategies $\sigma_i(h)$ and $\sigma_i'(h)$ in $\Gamma(h)$. We say that $\sigma_i(h)$ weakly dominates $\sigma_i'(h)$ in $\Gamma(h)$ if (1) $f_i^h((\sigma_i(h), \sigma_j(h))) \geq f_i^h((\sigma_i'(h), \sigma_j(h)))$ for all player j's strategies $\sigma_j(h)$ in $\Gamma(h)$, and (2) $f_i^h((\sigma_i(h), \sigma_j(h))) > f_i^h((\sigma_i'(h), \sigma_j(h))$ for at least one $\sigma_j(h)$ in $\Gamma(h)$. The second condition that Bhaskar imposed requires that if $\sigma = (\sigma_1, \sigma_2)$ is the equilibrium, then the following hold for both players $i = 1,2$: in each subgame $\Gamma(h)$, there is no strategy of player i which dominates $\sigma_i \mid h$ in $\Gamma(h)$ where $\sigma_i \mid h$ is the restriction of σ_i to the subgame $\Gamma(h)$.

In addition to the two conditions, we require equilibrium strategies to be Markov (or stationary) and conservative.

A player's strategy is called Markov if each action induced by the strategy depends only on a prevailing action combination. Thus player 1's (player 2's, resp.) Markov strategy is a function from $\{e\} \bigcup X$ to X_1 (from $X_1 \bigcup X$ to X_2, resp.). We will hereafter use ρ_1 and ρ_2 to denote Markov strategies of players 1 and 2.

The restriction to Markov strategies greatly simplifies the analysis since interactions of players' strategies are kept as simple as possible. But a more important reason for imposing the Markov property comes from one of the objectives of the paper; that is, the study of the $vN-M$ stable set or its variants in strategic form games from the viewpoint of equilibria in their extended games with preplays. Since the $vN-M$ stable set is a static solution concept, we want the stability being independent of the history of preplay negotiations.[3]

A mathematical justification of restricting to the Markov strategy was given in Harsanyi [1974, Lemmas 6 and 7]. That is, if $\rho = (\rho_1, \rho_2)$ is a Nash equilibrium when players are restricted to using the Markov strategies, then ρ is still an equilibrium even if each player is free to use any strategy in Σ_i (not necessarily Markov).

The conservativeness, initially defined by Harsanyi [1974], assumes that each player never moves unless he will positively benefit from this move. The assumption arises also from the study of the $vN - M$ stable set: it assumes such conservativeness in its definition. Formally the conservativeness is defined in the following manner. Take a strategy combination ρ^*, and a subgame $\Gamma(h^k)$ which follows the history $h^k = (x^1, x^2, \ldots, x^k)$ up to the kth period. ρ^* is called conservative in $\Gamma(h^k)$ if the following hold. Let z be the final outcome in $\Gamma(h^k)$ under the restriction of ρ^* to this subgame: z may be an infinite sequence of outcomes. Then (1) $z = x^k$ or (2) If $x^{k+1}, x^{k+2}, \ldots (i^{k+1}, i^{k+2}, \ldots$, resp.) is the sequence of outcomes (of corresponding players, resp.) under ρ^*, then

$$u_{i^l}(z) > u_{i^l}(x^{l-1}) \text{ for all } l = k+1, k+2, \ldots$$

except for $l = K$ or $K - 1$ where K is the period that the game ends.

Since payoffs are nonnegative and further in case the game never ends they are zero, the game must end after a finite number of steps if a pair of players' strategies is conservative.

A strategy combination $\rho = (\rho_1, \rho_2)$ is called a conservative Markov perfect equilibrium, denoted by CMPE hereafter, of the extended game if it satisfies the four conditions above, i.e.,

1. ρ is subgame perfect;
2. ρ_1, ρ_2 are not weakly dominated in each subgame;
3. ρ_1, ρ_2 are Markov strategies; and
4. ρ is conservative in each subgame.

[3]Other defenses of assuming the Markov property, in particular, in analyzing duopoly markets, are found in Maskin and Tirole [1988].

The restriction to Markov strategies makes it possible to describe subgames in a simpler way. That is, it is sufficient to make clear starting action combination x and player i to move first. Thus subgames will hereafter be denoted by $\Gamma(x,i), x \in X, i = 1,2$. $\Gamma(e,1)(\Gamma(e,2)$, resp.) is the whole extended game starting from the move of player 1 (player 2, resp.).

We say $\rho = (\rho_1, \rho_2)$ is a CMPE in the subgame $\Gamma(x,i)$ if ρ_i's are Markov and ρ satisfies (1),(2),(4) above in each subgame of $\Gamma(x,i)$.

4 Typical Two-Person Games and the CMPE

In this section, we apply the preplay to typical two-person games: the prisoner's dilemma, a pure coordination game and the battle of the sexes.

4.1 The Prisoner's Dilemma

Consider the following prisoner's dilemma G^1 and its extended games.

Example 4.1 (the prisoner's dilemma):

1 \ 2	C	D
C	4,4	0,5
D	5,0	1,1

Proposition 4.1: *Let ρ^* be a CMPE in $\Gamma(e,i), i = 1$ or 2. Then the following must hold.*

$$\rho_1^*(CC) = C, \quad \rho_1^*(CD) = D, \quad \rho_1^*(DC) = D, \quad \rho_1^*(DD) = D,$$
$$\rho_2^*(CC) = C, \quad \rho_2^*(CD) = D, \quad \rho_2^*(DC) = D, \quad \rho_2^*(DD) = D.$$

That is, player 1 (player 2, resp.) changes his action only at the outcome $CD(DC$, resp.). Figure 4.1 depicts ρ_1^*, ρ_2^* and the induced movements.

Therefore the subgames $\Gamma(CC,1)$ and $\Gamma(CC,2)$ end at the outcome CC; $\Gamma(CD,2)(\Gamma(DC,1)$, resp.) ends at $CD(DC$, resp.); and $\Gamma(CD,1), \Gamma(DC,2), \Gamma(DD,1)$ and $\Gamma(DD,2)$ end at DD.

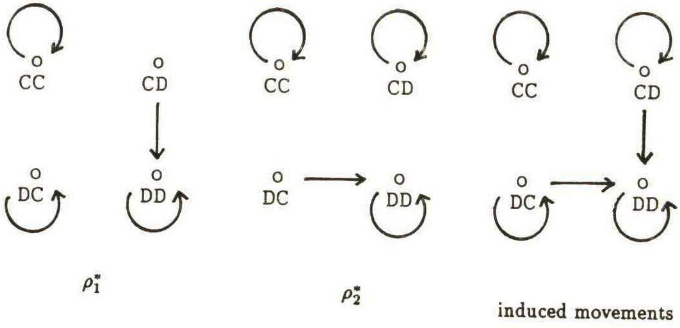

Figure 4.1: ρ_1^*, ρ_2^* and the induced movements

Proof: Take first the subgame $\Gamma(CD, 2)$. Since player 2 gains his maximal payoff 5 at CD, we must have $\rho_2^*(CD) = D$ by the conservativeness.[4] Similarly $\rho_1^*(DC) = D$ follows. Then by the subgame perfectness we have $\rho_1^*(DD) = D$ since player 1's payoff is 1 at DD but it is 0 at CD: if player 1 moves, the game ends at CD since $\rho_2^*(CD) = D$. Similarly $\rho_2^*(DD) = D$ holds. Next take $\Gamma(CD, 1)$. Then if player 1 moves, the game ends at DD since $\rho_2^*(DD) = D$. Since his payoffs are 0 at CD and 1 at DD, we must have $\rho_1^*(CD) = D$ by the subgame perfectness. Similarly $\rho_2^*(DC) = D$. Finally again by the subgame perfectness we obtain $\rho_1^*(CC) = \rho_2^*(CC) = C$. Q.E.D.

On the basis of Proposition 4.1, we may show that in the whole game every CMPE produces the unique outcome CC.

[4]This can be shown also by using the condition that no weakly dominated strategies be used in equilibria. As for the propositions concerning the three 2×2 games in this section, only one of these two conditions is necessary to prove them. In the price-setting duopoly to be studied in the next section, both conditions are necessary to prove main propositions.

Proposition 4.2: *Take the whole game $\Gamma(e, i), i = 1$ or 2. Then every CMPE in $\Gamma(e, i)$ induces CC as its final outcome.*

Proof: Take a CMPE ρ^* and a subgame starting from the second period. This subgame is $\Gamma(C, j)$ or $\Gamma(D, j), j \neq i$, depending on the player i's choice in the first period. Take first the game $\Gamma(C, j)$. Then, from Proposition 4.1, the following hold. If j chooses C, then the game ends at CC and j gets the payoff 4; while if he chooses D, then the game ends at DD with payoff 1. Thus we must have $\rho_2^*(C) = C$. Take next the game $\Gamma(D, j)$. In this case, the game ends at DD irrespective of player j's choice: recall Proposition 4.1. Thus $\rho_j^*(D) = C$ or D.

Finally take the game $\Gamma(e, i)$. If i chooses C, then the game ends at CC and he gets the payoff 4; while if he chooses D, then the game ends at DD where his payoff is 1. Thus $\rho_i^*(e)$ must be C.

Therefore every CMPE induces CC as its final outcome. Q.E.D.

Remark 4.1:

(1) It is easily seen from the discussions in the proofs of Propositions 4.1 and 4.2 that ρ^* constructed in these propositions are CMPE.

(2) There are two outcomes CC and DD which are stable in the sense that neither player has an incentive to move. In the whole game, however, only one of them, i.e., CC, is realized; and further this holds irrespective of the order of players' moves, i.e., who moves first. In games with continuous action spaces, however, the multiplicity of stable outcomes may cause the multiplicity of equilibria in the whole game. See Section 6 for details.

4.2 A Pure Coordination Game

Consider the following pure coordination game G^2 and its extended games.

Example 4.2 (a pure coodination game):

1 \ 2	L	R
U	2,2	0,0
D	0,0	1,1

Proposition 4.3: *Let ρ^* be a CMPE. Then the following must hold.*

$$\rho_1^*(UL) = U, \quad \rho_1^*(UR) = D, \quad \rho_1^*(DL) = U, \quad \rho_1^*(DR) = U,$$
$$\rho_2^*(UL) = L, \quad \rho_2^*(UR) = L, \quad \rho_2^*(DL) = R, \quad \rho_2^*(DR) = L.$$

That is, each player changes his action except when he is at UL. Figure 4.2 depicts ρ_1^*, ρ_2^* and the induced movements.

Therefore all subgames of the form $\Gamma(x,i), x = UL, UR, DL, DR,\ i = 1,2$, end at the outcome UL.

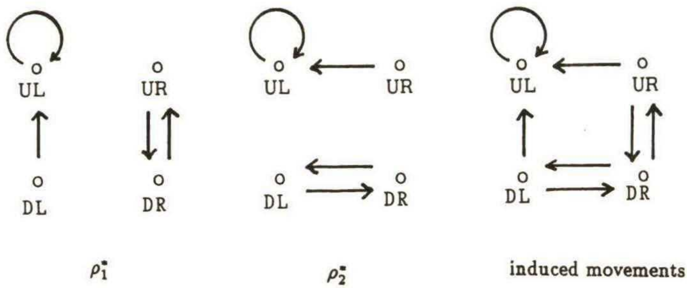

Figure 4.2: ρ_1^*, ρ_2^* and the induced movements

Proof: This proposition is proved in a similar manner to the proof of Proposition 4.1 by the use of the subgame perfectness and the conservativeness.

Proposition 4.4: *Take the whole game $\Gamma(e,i), i = 1$ or 2. Then every CMPE in $\Gamma(e,i)$ induces UL as its unique final outcome.*

Proof: This is clear from Proposition 4.3. We may take either L or R for player 2, and either U or D for player 1 in their choices in the first two periods. Q.E.D.

Remark 4.2: It is easily seen that the ρ^* constructed in the propositions above are CMPE.

4.3 The Battle of the Sexes

Consider the following battle of the sexes G^3 and its extended games.

Example 4.3 (the battle of the sexes:)

$1 \setminus 2$	L	R
U	2,1	0,0
D	0,0	1,2

Proposition 4.5: *Let ρ^* be a CMPE. Then the following must hold.*

$$\rho_1^*(UL) = U, \quad \rho_1^*(UR) = D, \quad \rho_1^*(DL) = U, \quad \rho_1^*(DR) = U,$$
$$\rho_2^*(UL) = R, \quad \rho_2^*(UR) = L, \quad \rho_2^*(DL) = R, \quad \rho_2^*(DR) = R.$$

That is, player 1 (player 2, resp.) chooses not to change his action only at UL (DR, resp.). Figure 4.3 depicts ρ_1^*, ρ_2^* and the induced movements.

Therefore the subgames $\Gamma(UL,1), \Gamma(DL,1), \Gamma(DR,1)$ and $\Gamma(UR,2)$ end at the outcome UL; and the subgames $\Gamma(UL,2), \Gamma(UR,1), \Gamma(DL,2)$, and $\Gamma(DR,2)$ end at DR.

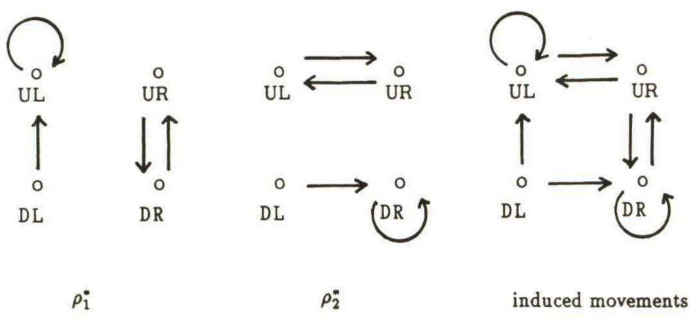

Figure 4.3: ρ_1^*, ρ_2^* and the induced movements

Proof: This proposition is proved in a similar manner to the proof of Proposition 4.1.

The next proposition shows that final outcomes induced by CMPE may depend on which player moves first.

Proposition 4.6: *In the whole game $\Gamma(e,1)$, every CMPE induces the outcome DR; while in $\Gamma(e,2)$, every CMPE induces UL.*

Proof: Take any CMPE ρ^*. First consider the game $\Gamma(e,1)$. Take the subgame $\Gamma(U,2)$ starting from the second period. If player 2 chooses L, then the game ends at UL since $\rho_1^*(UL) = U$: player 2 gets 1 at UL. If he chooses R, then the play proceeds $UR \to DR$ and stops at DR since $\rho_1^*(UR) = D$ and $\rho_2^*(DR) = R$: player 2 gets 2. Hence $\rho_2^*(U) = R$.

Take next the subgame $\Gamma(D,2)$. If player 2 chooses L, then the play proceeds $DL \to UL \to UR \to DR$ and stops at DR; while if he chooses R, then the play proceeds $DR \to UR \to UL$ and stops at UL. Player 2 gains the payoff 1 at UL and 2 at DR. Thus we must have $\rho_2^*(D) = L$.

Finally take the whole game $\Gamma(e,1)$. If player 1 chooses U, then the game ends at DR. Even if he chooses L, the game ends also at DR. Thus every CMPE induces DR as its final outcome.

The latter half of the proposition is similarly proved. Q.E.D.

Remark 4.3: The ρ^* defined in the propositions are easily shown to be CMPE.

4.4 Stability and Semi-Stability of Outcomes

The three examples show that in CMPE there may exist two types of final outcomes. The outcomes CC in the prisoner's dilemma and UL in the pure coordination are of the first type; and the outcomes UL and DR in the battle of the sexes are of the second type. In the former, neither player has an incentive to deviate; while in the latter only one player who is to move prefers not to move: another player has an incentive to move. Hereafter the former type of outcome will be called stable; and the latter type semi-stable. Symbols $K(\rho)$ and $K'(\rho)$ will be used to denote the sets of these stable outcomes, i.e.,

$$K(\rho) = \{x \in X : \rho_i(x) = x_i \text{ for } i = 1,2\}$$

and

$$K'(\rho) = \{x \in X : \rho_i(x) = x_i \text{ and } \rho_j(x) \neq x_j\}.$$

In each subgame, every final outcome induced by a CMPE ρ must be stable or semi-stable, i.e., must be in $K(\rho) \bigcup K'(\rho)$. It is not necessary, however, that every stable outcome is a final outcome: recall the outcome DD in the prisoner's dilemma. Further the set $K(\rho)$ may be empty as shown in the battle of the sexes. Later in Section 7 relations between stable outcomes and the $vN - M$ stable set in the action combination space will be studied.

5 A Price-Setting Duopoly and the CMPE

In this section we study the CMPE in a price-setting duopoly. We assume the following symmetric duopoly. Two firms 1,2 are producing homogeneous goods with the same marginal cost c. For simplicity, let $c = 0$ in what follows. Consumers' demands are represented by a demand function $D(p)$. $D(p)$ is decreasing in p, and there exists a price $\tilde{p}$ such that $D(p) = 0$ for all $p \geq \tilde{p}$. The market profit at price p is $\pi(p) = pD(p)$, and $\pi(0) = \pi(\tilde{p}) = 0$. Suppose $\pi(p)$ is continuous and strictly concave. Then there is a unique price p^m, called the monopoly price, which maximizes $\pi(p)$. Denote firm 1's (firm 2's, resp.) price level by $p^1 (p^2$, resp.). If their prices are equal, they split even the market profit; otherwise all sales go to a lower pricing firm. This duopoly market is written as the following two-person game:

$$G^B = (N = \{1,2\}, \{X_i\}_{i=1,2}, \{u_i\}_{i=1,2})$$

where $X_i = [0, \tilde{p}]$ for $i = 1,2$,

and

$$u_i : X = X_1 \times X_2 \rightarrow R_+ \text{ (nonnegative reals) defined by}$$

$$u_i(p_i, p_j) = \begin{cases} \pi(p_i) & \text{if } p_i < p_j \\ \pi(p_i)/2 & \text{if } p_i = p_j \quad \text{for } i,j = 1,2,\ i \neq j \\ 0 & \text{if } p_i > p_j. \end{cases}$$

Throughout this section, we assume that player (firm) 1 is first to move. But, needless to say,the same results hold even if player 2 moves first because of their symmetry.

Bhaskar [1989] showed that every perfect equilibrium of the whole extended game of G^B produces the unique outcome (p^m, p^m), i.e., the pair of the monopoly prices. Here the perfect equilibrium is a subgame perfect equilibrium that has no weakly dominated strategy in any subgame; but neither the Markov property nor the conservativeness is assumed.[5] In the following, we will show that in every CMPE the monopoly price pair can be reached in every subgame, i.e., the monopoly price pair can be reached even if we start preplays with an arbitrarily chosen price pair.

Remark 5.1: The existence of a CMPE in which the monopoly price can be reached from an arbitrary chosen price pair was already shown in Bhaskar [1989, Proposition 1]. Though he did not assume the Markov property, the strategies that he constructed in the proposition are Markov.

Before presenting the main results, we first prove a sequence of lemmas. The $\rho = (\rho_1, \rho_2)$ which will appear in the lemmas is an arbitrary chosen CMPE of $\Gamma(e, 1)$.

Lemma 5.1: [6] *Take a subgame $\Gamma(p^0, i), p^0 \in X, i = 1$ or 2. Let p^* be the final outcome induced by ρ in the subgame. Then the following hold.*

1. *If $p^* = p^0$, then we never have $0 < p_j^0 < p_i^0$.*

2. *If $p^* \neq p^0$, then neither $0 < p_j^* < p_i^*$ nor $0 < p_i^* < p_j^*$ holds.*

Proof:

1. Suppose $0 < p_j^0 < p_i^0$ holds. Since the subgame ends at p^0, player i gets the payoff 0. Define ρ_i' by

$$\rho_i'(p) = \begin{cases} p_j & \text{if } p_i > p_j \\ p_i & \text{otherwise.} \end{cases}$$

Then it gives player 1 at least payoff 0 in $\Gamma(p^0, i)$. Further it gives him a positive payoff

[5] As far as I see, the two conditions that Bhaskar imposed on equilibrium are not enough to show the uniqueness of the equilibrium outcome. More conditions, say the conservativeness, seem to be necessary.

[6] This lemma corresponds to Proposition 2a of Bhaskar [1989]. The Bhaskar's two conditions seem to be insufficient to prove the lemma.

if player j uses the same sort of strategy ρ'_j, i.e.,

$$\rho'_j(p) = \begin{cases} p_i & \text{if } p_j > p_i \\ p_j & \text{otherwise.} \end{cases}$$

Thus ρ_i is weakly dominated by ρ'_i, a contradiction.

2. Without loss of generality, suppose $\rho_1(p^*) = p_1^*$ and the game ends at player 1's turn. If $0 < p_2^* < p_1^*$, then a contradiction follows in the same manner as in 1. Suppose $0 < p_1^* < p_2^*$. Then player 2' payoff is 0. Since $p^* \neq p^0$ and the game ends at 1's turn, we must have $p' = (p'_1, p'_2)$ satisfying the following:

$$p'_1 = p_1^* \text{ and } p'_2 \neq p_2^*; \text{ and}$$

$$\text{player 2 is the player to move at } p'.$$

The conservativeness in $\Gamma(p', 2)$ implies that in the final outcome p^* player 2 must get more than in p'. Thus we have a contradiction since player 2 gets 0 at p^* and gets at least 0 in p'.
Q.E.D.

Remark 5.2: Lemma 5.1 shows that under any CMPE if a subgame $\Gamma(p, i)$ ends at p^* different from p, then $p_i^* = p_j^*$ or $min(p_i^*, p_j^*) = 0$. In the former case, each firm gets the highest payoff when $p^* = (p^m, p^m)$; and in the latter each firm gets 0.

Lemma 5.2: $\rho_1(p^m, p^m) = \rho_2(p^m, p^m) = p^m$.

Proof: This is clear from Remark 5.2 and the conservativeness of ρ in $\Gamma((p^m, p^m), i)$, $i = 1, 2$.

Before stating the next lemma, we introduce notation p^{m-} and p^{m+}. p^{m-} (p^{m+}, resp.) is the price less than (greater than, resp.) p^m which gives a lower pricing firm the half of the monopoly profit; i.e., p^{m-} and p^{m+} satisfy $\pi(p^{m-}) = \pi(p^{m+}) = \pi(p^m)/2$ and $p^{m-} < p^m < p^{m+}$. Since π is continuous and strictly concave and $\pi(0) = \pi(\tilde{p}) = 0$, the existence and the uniqueness of p^{m-} and p^{m+} follow.

Remark 5.3: The following fact is also easily seen. When $p_i < p_j, u_i(p_i, p_j) > u_i(p^m, p^m)$ holds if and only if $p^{m-} < p_i < p^{m+}$.

Lemma 5.3:

1. *For every $p = (p_1, p^m), 0 \leq p_1 < p^{m-}$ or $p^m < p_1$, the final outcome under ρ in the subgame $\Gamma(p, 1)$ is (p^m, p^m).*

2. *For every $p = (p^m, p_2), 0 \leq p_2 < p^{m-}$ or $p^m < p_2$, the final outcome under ρ in the subgame $\Gamma(p, 2)$ is (p^m, p^m).*

Proof:

1. By Lemma 5.2, if player 1 chooses p^m, then the game ends and he gets $u_i(p^m, p^m) = \pi(p^m)/2$. Thus the final outcome under ρ must guarantee player 1 at least the payoff $u_i(p^m, p^m)$. By Remark 5.2, thus the final outcome must be (p^m, p^m). Note that if $0 < p_1 < p^{m-}$ or $p^m < p_1$, player 1 gets less than $u_i(p^m, p^m)$ when he chooses not to move at p.

2. is similarly shown. Q.E.D.

Lemma 5.4:

1. *For every $p = (p_1, 0)$, the final outcome under ρ in the subgame $\Gamma(p, 1)$ is (p^m, p^m).*

2. *For every $p = (0, p_2)$, the final outcome under ρ in the subgame $\Gamma(p, 2)$ is (p^m, p^m).*

Proof:

1. Suppose player 1 chooses p^m in his first move in $\Gamma(p, 1)$. Then the subgame $\Gamma((p^m, 0), 2)$ follows. Then by Lemma 5.3 the final outcome of $\Gamma((p^m, 0), 2)$ will be (p^m, p^m). By Remark 5.2, the final outcome of $\Gamma(p, 1)$ must be (p^m, p^m).

2. is similarly shown. Q.E.D.

Lemma 5.5: *Take a price pair $p = (p_1, p_2), p_1 \neq p_2$. Suppose $p_i < p_j$. Then the final outcome under ρ in $\Gamma(p, j)$ is (p^m, p^m).*

Proof: Suppose player j chooses price 0 in his first move in $\Gamma(p, j)$. Then the subgame $\Gamma((p_1, 0), i)$ follows. By Lemma 5.4 the final outcome of $\Gamma((p_i, 0), i)$ will be (p^m, p^m). By Remark 5.2 the final outcome of $\Gamma(p, j)$ must be (p^m, p^m). Q.E.D.

Lemma 5.6: *For each (p, p), $p \neq p^m$, the final outcome under ρ in $\Gamma((p, p), i), i = 1, 2$, is (p^m, p^m).*

Proof: Take any $(p, p), p \neq p^m$, and take the subgame $\Gamma((p, p), i)$. Suppose player i chooses 0. Then the subgame $\Gamma((0, p), 2)$ follows. Thus by Lemma 5.4 the final outcome must be (p^m, p^m). Hence by Remark 5.1 the final outcome in $\Gamma((p, p), i)$ must be (p^m, p^m). Q.E.D.

Lemma 5.7: *Take a subgame $\Gamma(p, i)$ with $p = (p_1, p_2), p^{m-} \leq p_i \leq p^{m+}, p_i < p_j$. Then $\rho_i(p) = p_i$. Thus player i chooses not to move, and the game ends at p.*

Proof: If i does not move, the game ends and he gains $u_i(p_1, p_2) \geq u_i(p^m, p^m)$. If he moves at p, he will get at most $u_i(p^m, p^m)$ in the final outcome of $\Gamma(p, i)$: recall Remark 5.2. Thus by the conservativeness player i does not move. Q.E.D.

From these lemmas, the proposition follows which shows that under every CMPE, the following holds: in each price pair other than the pair of the monopoly prices, at least one player has an incentive to move, and his move induces a sequential movement of prices which eventually reaches the monopoly price pair.

Proposition 5.8: *Let $\rho = (\rho_1, \rho_2)$ be a CMPE of $\Gamma(e, 1)$ and take a price pair (p_1, p_2). Then the subgames $\Gamma((p_1, p_2), i), i = 1, 2$, has the final outcome (p^m, p^m) under ρ, except when $p^{m-} \leq p_i \leq p^{m+}$ and $p_i < p_j$; and if this is the case the subgame ends at (p_1, p_2).*

Proof: The claim easily follows from Lemmas 5.1 - 5.7.

Remark 5.4: It follows also from the lemmas that under every CMPE ρ the monopoly price pair (p^m, p^m) is the unique stable outcome, and price pairs (p_1, p_2) satisfying $p^{m-} \leq p_i \leq p^{m+}$ and $p_i < p_j, i, j = 1, 2$, are semi-stable outcomes.

We now pick up the first two periods of the game, and examine players' choices. The next proposition is for player 2's decision in the second period.

Proposition 5.9: *Let $\rho = (\rho_1, \rho_2)$ be a CMPE of $\Gamma(e, 1)$. Take p_1 with $p^{m-} \leq p_1 \leq p^{m+}$. Then $\rho_2(p_1) \leq p_1$ must hold, and the subgame $\Gamma(p_1, 2)$ has the final outcome (p^m, p^m) under ρ.*

Proof: Suppose $p_2 = \rho_2(p_1) > p_1$. Then Lemma 5.7 shows that $\rho_1(p_1, p_2) = p_1$ and the game ends at (p_1, p_2). Player 2 gets 0. If $p_2 = \rho_2(p_1) \leq p_1$, then Proposition 5.8 shows that the final outcome in the subgame $\Gamma((p_1, p_2), 1)$ under ρ is (p^m, p^m) in which player 2 gets $\pi(p^m)/2 > 0$. Thus $\rho_2(p_1) \leq p_1$ must hold. Q.E.D.

Proposition 5.10: *Let $\rho = (\rho_1, \rho_2)$ be a CMPE of $\Gamma(e, 1)$. Then player 1's choice $\rho_1(e)$ in the first period can be arbitrary; and the game ends at the monopoly price pair (p^m, p^m) irrespective of his choice.*

Proof: This is clear from Propositions 5.8 and 5.9.

6 The Prisoner's Dilemma with Continuous Action Spaces - A Folk Theorem

So far we have shown that the alternating-move preplay process works well in various examples. It succeeds in reaching an outcome that is favorable to both players: (C,C) in the prisoner's dilemma, the Pareto efficient outcomes in the pure coordination and in the price-setting duopoly game.

The following discussion, however, reveals that the preplay does not always lead to such outcomes. It will be shown that in the prisoner's dilemma with continuous action spaces, every individually rational outcome could be attained as a final outcome of a CMPE: a sort of Folk theorem holds.

Example 6.1: $G^{PD} = (N = \{1,2\}, \{X_i\}_{i=1,2}, \{u_i\}_{i=1,2})$ where

$$X_i = [0,1], \text{ the closed interval between 0 and 1, } i = 1,2,$$
$$u_1(x_1, x_2) = 1 - x_1 + 4x_2, \text{ and}$$
$$u_2(x_1, x_2) = 1 - x_2 + 4x_1 \text{ for } (x_1, x_2) \in X = X_1 \times X_2.$$

The game is the mixed extension of the prisoner's dilemma given in Example 4.1

As for the payoffs, we state the following simple but useful facts without proofs.

Fact 6.1: For any fixed x_i, $u_j(x_1, x_2)$ is decreasing in x_j, for $i,j = 1,2, i = j$. Thus it is easily seen that $x_i = 0$ is the player i's maxmin action, $i = 1,2$, and that $(x_1, x_2) = (0,0)$ is the unique Nash equilibrium.

In what follows we call outcomes (x_1, x_2) with $u_i(x_1, x_2) \geq 1 (= u_i(0,0)), i = 1,2$, individually rational.

Fact 6.2: For each (x_1, x_2), take the line with slope $\frac{1}{4}$ which passes through (x_1, x_2). Then player 1's payoff in each outcome on the line is the same as $u_1(x_1, x_2)$; and his payoff is bigger (smaller, resp.) than $u_1(x_1, x_2)$ if an outcome is above (below, resp.) the line. For player 2, take the line with slope 4. Then his payoff is the same as $u_2(x_1, x_2)$ in each outcome on the line; and it is bigger (smaller, resp.) than $u_2(x_1, x_2)$ if an outcome is to the right (left, resp.) of the line. See Figure 6.1.

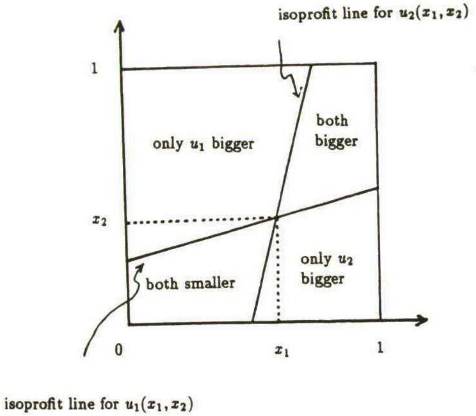

Figure 6.1: Isoprofit lines

We now take any monotone and continuous curve connecting $(0,0)$ (the point O) with a point on the edge EC or the edge DC where $D = (1, \frac{1}{4})$ and $F = (\frac{1}{4}, 1)$. Thus $OD(OE$, resp.) is the isoprofit line of $u_1(0,0) = 1$ ($u_2(0,0) = 1$, resp.); and for every outcome (x_1, x_2) in $ODCE, u_1(x_1, x_2), u_2(x_1, x_2) \geq 1$. See Figure 6.2. The monotonicity implies that each player's payoff never decreases as we move from $(0,0)$ to the other end point. Note that each outcome on the line $OD(OE$, resp.) gives player 1 (player 2, resp.) the constant payoff 1. Define $\rho = (\rho_1, \rho_2)$ as follows.

1. If (x_1, x_2) is on the curve, then $\rho_i(x_1, x_2) = x_i, i = 1, 2$;

2. if (x_1, x_2) is to the right of the curve, then $\rho_1(x_1, x_2) = x_1', \rho_2(x_1, x_2) = x_2$; and

3. if (x_1, x_2) is above the curve, then $\rho_1(x_1, x_2) = x_1, \rho_2(x_1, x_2) = x_2'$.

Here $x_1'(x_2'$, resp.) is the value such that $(x_1', x_2)((x_1, x_2')$, resp.) is on the curve. See Figure 6.2. Such x_1' and x_2' are uniquely determined because of Fact 6.1 and the monotonicity of the curve. Then the following proposition holds.

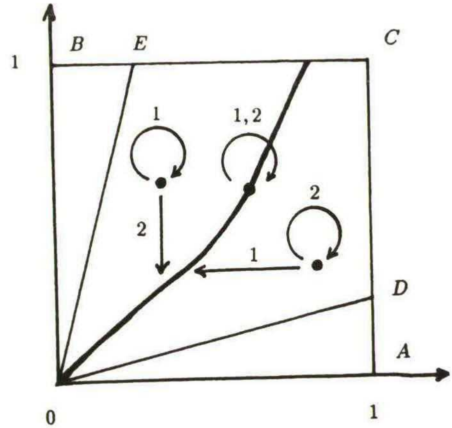

Figure 6.2: Illustration of ρ_1 and ρ_2

Proposition 6.3: The ρ defined above is a CMPE for every subgame $\Gamma(x,i), x \in X$ and $i = 1,2$.

Proof: It is clear from Facts 6.1 and 6.2.

We now show the main result of this section.

Proposition 6.4: *For each $x = (x_1, x_2) \in X$ with $u_1(x), u_2(x) \geq 1$, there exists at least one CMPE ρ which attains x as its unique final outcome in the whole game $\Gamma(e,1)$.*

Proof: Take any action combination $x^* = (x_1^*, x_2^*) \in X$ with $u_i(x^*) \geq 1, i = 1,2$. Take the isoprofit line (w.r.t. player 1) FG passing through x^*. See Figure 6.3. The outcome G (F, resp.) is on the edge ECD (on the line OE, resp.) and the line FG has the slope $\frac{1}{4}$; recall Fact 6.2. The points F and G are $((-x_1^* + 4x_2^*)/15, 4(-x_1^* + 4x_2^*)/15)$ $(1, (1 - x_1^* + 4x_2^*)/4)$, respectively. It is easily observed that when we go from the outcome O to the outcome G via F, player 2's payoff is constant and 1's payoff increases on OF, while on FG player 1's payoff is constant and 2's payoff increases. Thus the OFG is a monotone and continuous curve connecting the outcome O and the outcome G on the edge ECD. Therefore by Proposition 6.3 the $\rho = (\rho_1, \rho_2)$ defined below is a CMPE for each subgame of the form $\Gamma(x,i), x \in X, i = 1,2$. For each $x = (x_1, x_2)$,

1. if (x_1, x_2) is on OFG, then $\rho_i(x_1, x_2) = x_i, i = 1, 2$;

2. if (x_1, x_2) is to the right of OFG, then $\rho_1(x_1, x_2) = x_1', \rho_2(x_1, x_2) = x_2$; and

3. if (x_1, x_2) is above OFG, then $\rho_1(x_1, x_2) = x_1, \rho_2(x_1, x_2) = x_2'$,

where x_1' and x_2' are the outcomes such that (x_1', x_2) and (x_1, x_2') are on OFG.

Take now the subgame $\Gamma(x_1, 2)$ starting from the 2nd period. Then for each $x_1 \in X_1, \rho_2(x_1) = x_2$ where x_2 is such that (x_1, x_2) is on OFG constitutes a CMPE in the subgame $\Gamma(x_1, 2)$ together with the ρ satisfying 1, 2 and 3 above. In fact, take x_2' such that (x_1, x_2') is above OFG, then by the definition of $\rho_1(x_1, x_2')$ player 1 chooses not to move; and the game ends at (x_1, x_2'). Player 2 gets less than the payoff at (x_1, x_2) on OFG: recall Fact 6.1. Suppose next (x_1, x_2') is to the right of OFG. Then again from the definition of $\rho_1(x_1, x_2')$ player 1 changes x_1 to x_1' such that (x_1', x_2') is on OFG; and then the game ends. Player 2's payoff at (x_1', x_2') is clearly not more than that at (x_1, x_2). See Figure 6.3. Therefore $\rho_2(x_1) = x_2$ such that (x_1, x_2) is on OFG constitutes a CMPE together with ρ above.

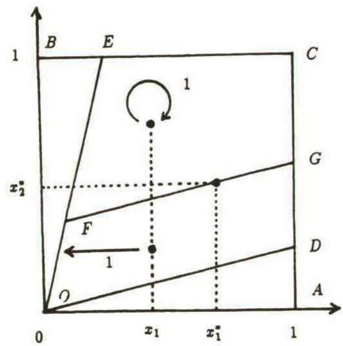

Figure 6.3: Illustration of $\rho_2(x_1)$

Finally take the whole game $\Gamma(e, 1)$. Then under $\rho = (\rho_1, \rho_2)$ defined above, for each $x_1 \in X_1$, player 2 takes x_2 such that (x_1, x_2) is on OFG, and then the game ends. By the definition of OFG, player 1's payoff is constant on the line FG, which is larger than his payoff on OF. Thus if $\rho_1(e) = x_1, (-x_1^* + 4x_2^*)/15 < x^1 < 1$, then this $\rho_1(e)$ constitutes a CMPE in the whole game together with the ρ_1, ρ_2 defined above. In particular, we can obtain the desired one by letting $\rho_1(e) = x_1^*$. Q.E.D.

Remark 6.1: Proposition holds also for $\Gamma(e,2)$, the game in which player 2 first moves. Take the isoprofit line for player 2; then similar arguments follow.

The next proposition shows that there exists no CMPE that induces an outcome in which u_1 or u_2 is strictly less than 1, i.e., an outcome not being individually rational. Such outcomes are in the regions OAD or OBE in Figure 6.1. We first prove a lemma.

Lemma 6.5: *For every CMPE ρ of $\Gamma(e,i)$, $i = 1,2$, we must have $\rho_1(0,0) = \rho_2(0,0) = 0$.*

Proof: Suppose w.l.o.g. $\rho_1(0,0) = x_1' \neq 0$. Take the subgame $\Gamma((0,0),1)$ and suppose the game ends at $x^* = (x_1^*, x_2^*)$ under ρ. By the conservativeness, the subgame ends after a finite number of steps, and further $u_1(x^*) > u_1(0,0) = 1$. Since $u_1(x_1, 0) < 1$, player 2 must be involved in the path from $(0,0)$ to x^*. Let the path of outcomes from $(0,0)$ to x^* and the corresponding sequence of players be $(0,0) = x^0, x^1, \ldots, x^{k-1}, x^k = x^*$ and $i^1, \ldots, i^k$, respectively. Note that $i^1 = 1$ and $k \geq 2$. Suppose first $i^k = 1$; and the game ends in player 2's turn. Then from Fact 6.1 we must have $x_1^* = x_1^k < x_1^{k-1}$. If otherwise, player 1 chooses not to move at x^{k-1}. Then the game ends at x^{k-1} and he gets more payoff, which contradicts that ρ is a CMPE. Since $i^{k-1} = 2$, we have $x_2^{k-2} > x_2^*$ by the same reason. Thus we never reach $(0,0)$, which implies that we never have such a path from $(0,0)$ to x^*. When $i^k = 2$, a similar proof applies. Q.E.D.

Proposition 6.6: *For every CMPE ρ of $\Gamma(e,i), i = 1,2$, each player gets a payoff greater than or equal to 1 in the final outcome.*

Proof: Suppose w.l.o.g. there is a CMPE ρ such that $u_1(x) < 1$ holds in the final outcome x of the whole extended game. Suppose first the game ends in player 1's turn; hence, $\rho_1(x) = x_1$. Take the subgame $\Gamma(x,1)$. Then the subgame ends at x and player 1 gets the payoff $u_1(x) < 1$.

Suppose he chooses 0 instead. Then the subgame $\Gamma((0,x_2),2)$ follows. If $\rho_2(0,x_2) = 0$, then by Lemma 6.5 the game ends at $(0,0)$. Player 1 can thus increase his payoff, a contradiction.

Suppose $\rho_2(0,x_2) = x_2' \neq 0$. Since player 2 can get the payoff 1 by choosing 0, the final outcome must guarantee him a payoff greater than or equal to 1. Since his payoff at $(0,x_2')$ is less than 1, player 1's move must follow. Thus the final outcome must guarantee player 1 a payoff greater than $u_1(0, x_2')$ which is greater than 1. Hence player 1 can increase his payoff also in this case, a contradiction.

Suppose next the game ends in player 2's turn. Then we must have (x_1', x_2) such that $\rho_1(x_1', x_2) = x_1$. Take the subgame $\Gamma((x_1', x_2), 1)$; then a similar proof applies. Q.E.D.

7 Final Outcomes under CMPE and Stable Sets

In this section, we examine relations between stable outcomes under CMPE and the stable sets originally defined by von Neumann and Morgenstern [1953], hereafter denoted by $vN - M$ stable set, in the outcomes space, i.e., in the space of action pairs.

Similarly to Greenberg [1990], Chwe [1992] and Muto and Okada [1992], we define a binary relation, called the dominance relation, on the outcome space in the following manner. Take two outcomes $x = (x_1, x_2)$ and $y = (y_1, y_2) \in X$. We say that x is induced from y by player i, denoted $x \leftarrow|_i y$, if $x_j = y_j$, for $i, j = 1, 2, i = j$.

Definition 7.1 (Domination): For $x, y \in X$ and player $i = 1, 2$, x dominates y via i, denoted by $x dom_i y$ if (1) $x \leftarrow|_i y$ and (2) $u_i(x) > u_i(y)$. We simply say x dominates y, denoted $x dom y$, if $x dom_1 y$ or $x dom_2 y$.

Definition 7.2 (The $vN - M$ stable set w.r.t. *dom*): A set $V \subseteq X$ is a stable set w.r.t. *dom* if the following two conditions are satisfied. (1) For any two outcomes x, y in V, neither $x dom y$ nor $y dom x$; and (2) for any z not in V, there exists $x \in V$ such that $x dom z$. (1) and (2) are called internal and external stability, respectively.

Muto and Okada [1992] applied the $vN - M$ stable set w.r.t. *dom* to the price-setting duopoly; and they showed that unreasonable outcomes may be included in the stable set. They claimed that, to remove out these outcomes, one must take into account not only a direct domination but also a sequence of players' reactions that may ensue after a player changes his action. Harsanyi [1974] already pointed out the necessity of this indirect domination in the context of cooperative characteristic function form games. On the basis of Harsanyi's idea, we define the following indirect dominance relation on the outcome space.

Definition 7.3 (Indirect domination): For $x, y \in X, x$ indirectly dominates y, denoted by $xidomy$, if there exist a sequence of pairs of actions $y = x^0, x^1, \ldots, x^m = x$ and the corresponding sequence of players $i^1, \ldots, i^m$ such that for all $k = 1, 2, \ldots, m, i^k \neq i^{k-1}, x^k \leftarrow|_{i^k} x^{k-1}$ and $u_{i^k}(x) > u_{i^k}(x^{k-1})$.

Since there may exist various sequences of action pairs, Harsanyi proposed to pick up a particular one which may be supported by an equilibrium of an appropriately constructed noncooperative bargaining game.[7] The game models players' negotiation on how to distribute the amount that the grand coalition can gain. In parallel with the Harsanyi's approach, we consider the extended game with preplays, and pick up a particular sequence of indirect domination which is supported by a CMPE.[8]

Definition 7.4 (Effective domination): Take a CMPE ρ of the extended game $\Gamma(e, 1)$ or $\Gamma(e, 2)$. For $x, y \in X, x$ effectively dominates y under ρ, denoted $xedom(\rho)y$, if (1) $xdomy$, or (2) $xidomy$ with a sequence of action pairs $y = x^0, x^1, \ldots, x^m = x$ and a sequence of players $i^1, \ldots, i^m$ such that $x^k = \rho_{i^k}(x^{k-1})$ for $k = 2, \ldots, m$.

Definition 7.5(Effectively stable set) A set $V(\rho) \subseteq X$ is an effectively stable set under ρ if the following two conditions are satisfied. (1) For any two outcomes x, y in $V(\rho)$, neither $xedom(\rho)y$ nor $yedom(\rho)x$; and (2) for any z not in $V(\rho)$, there exists $x \in V(\rho)$ such that $xedom(\rho)z$. (1) and (2) are called internal effective stability and external effective stability, respectively.

It is easily seen that in the games studied in this paper, except the battle of the sexes, the sets $K(\rho)$ of stable outcomes under CMPE ρ are effectively stable sets: $\{CC, DD\}$ in the prisoner's dilemma, $\{UL\}$ in the pure coordination game, the monopoly price pair in the price-setting duopoly, and each of the monotone continuous curves in the continuous prisoner's dilemma.

In general, $K(\rho)$ always satisfies the internal effective stability as the next proposition shows.

[7] Another approach to resolve the multiplicity of sequences is to take the pessimistic or the optimistic one to the initial deviator. See Chwe [1992].

[8] The Harsanyi's bargaining game assumes the existence of a chairman who designates a coalition to propose a new offer. We do not take this approach since we would like to avoid the difficulty who pays the chairman's payoff.

Proposition 7.1: *Let ρ be a CMPE, and take the set $K(\rho)$ of its stable outcomes under ρ: $K(\rho)$ is the set of action pairs in which neither player moves under ρ. Recall the definition in Section 4.4. Then $K(\rho)$ satisfies the effective internal stability.*

Proof: Take two outcomes $x, y \in K(\rho)$, and suppose $x edom(\rho) y$. Then (1) $x dom y$ or (2) $x idom y$ with a sequence of outcomes $y = x^0, x^1, \ldots, x^m = x$ and a corresponding sequence of players $i^1, \ldots, i^m$ such that $x^k = \rho_{i^k}(x^{k-1})$ for $k = 2, \ldots, m$. Suppose (1) is the case. W.l.o.g. let $x dom_1 y$. Then $x_2 = y_2$ and $u_1(x) > u_1(y)$. Since $x \in K(\rho), \rho_i(x) = x_i, i = 1, 2$. Thus player 1 can improve his payoff by changing $\rho_1(y)$ from y_1 to x_1, contradicting that ρ is a CMPE. A similar proof is applied to the case (2). Q.E.D.

Recall in the battle of the sexes of Section 4.3, $K(\rho)$ is empty. Thus $K(\rho)$ does not satisfy the external stability. One sufficient condition for $K(\rho)$ to satisfy the external effective stability is given in the next proposition.

Proposition 7.2: *Let ρ be a CMPE and $K(\rho)$ be the set of its stable outcomes under ρ. Suppose there is no sequence (cycle) of outcomes $x^0, x^1, \ldots, x^m = x^0$ such that $x^k = \rho_{i^k}(x^{k-1})$ for $k = 1, \ldots, m$, $i^k \neq i^{k-1}, k = 1, \ldots, m-1$, and $i^1 = i^m$. Then $K(\rho)$ satisfies also external effective stablity, and thus it is an effectively stable set.*

Proof: Take any $z = (z_1, z_2) \notin K(\rho)$. W.l.o.g. assume $\rho_1(z) \neq z_1$. Since ρ is a CMPE, there exists a final outcome $x = (x_1, x_2)$ of the subgame $\Gamma(z, 1)$. Let the sequence of outcomes from z to x under ρ be $z = x^0, x^1, \ldots, x^m = x$; and let the corresponding sequence of players be $1 = i^1, \ldots, i^m$. Then $x edom(\rho) z$ because of the conservativeness of ρ. Thus if $x \in K(\rho)$, then the proof is done. Suppose $x \notin K(\rho)$. Suppose first $i^m = 1$. Then the game ends at player 2's turn and $\rho_2(x) = x_2$. Since $x \notin K(\rho)$, we have $\rho_1(x) \neq x_1$. Let $\rho_1(x) = y_1$ and let $x^{m+1} = (y_1, x_2)$. Then by the condition mentioned in the proposition $x^{m+1} \neq x^k$ for even k's. Furthermore, we never have $x^{m+1} = x^k$ for odd k's. In fact, if it is the case, the subgame $\Gamma(x^m, 1)$ ends at x^m; and thus $\rho_1(x) \neq x_1$ contradicts the conservativeness of ρ_1. Thus we obtain $x^{m+1} \neq x^k$ for $k = 0, 1, \ldots, m$. If player 1 changes $\rho_1(x^{m-1})$ from x_1 to y_1, then he can improve his payoff in the subgame $\Gamma(x^{m-1}, 1)$ contradicting that ρ is a CMPE. In fact, let x^*

be the final payoff under ρ in the subgame $\Gamma(x^{m+1}, 2)$. Then by the conservativeness we have $u_1(x^*) > u_1(x^m)$. A similar proof applies when $i^m = 2$. Q.E.D.

Remark 7.1: In the battle of the sexes, there exist cycles $DR \to UR \to DR$ w.r.t. player 1 and $UL \to UR \to UL$ w.r.t. player 2. Recall Figure 4.3.

8 Concluding Remarks

We have studied the validity of the alternating-move preplay process proposed by Bhaskar [1989] in general two-person games. The preplay works well in the prisoner's dilemma, a pure coordination game, and a price-setting duopoly game. It has been shown, however, that in the mixed extension of the prisoner's dilemma a sort of the Folk Theorem holds. We have further examined relations between the $vN - M$ stable set in strategic form games and equilibria in their extension with preplays. It has been shown that the set of outcomes, in which neither player has an incentive to deviate in equilibrium, always satisfies the internal effective stability; but it satisfies the external effective stability only when some conditions hold.

We conclude the paper with the following questions related to this research which merit future studies.

The first question concerns the assumption that players get zero payoff when the game goes on indefinitely. Though it was used in previous studies, Bhaskar [1989], Harsanyi [1974], etc., the assumption is rather artificial. One way to avoid the assumption is to introduce a small probability that the game terminates at the end of each period of preplays. Then the game ends in a finite number of steps with probability one; and expected payoffs can be calculated for all strategy combinations.

The second question also concerns the rule of preplays. The question is whether or not one may find a preplay process which invalidates the Folk Theorem in the continuous prisoner's dilemma. One possible way is to change the stopping rule so that the preplay ends only when both players choose not to move. Another possibility is to introduce simultaneous moves like the one proposed by Kalai [1981]. As we saw in the battle of the sexes, final outcomes may depend on which player moves first in the alternating-move preplays. Simultaneous-move preplays may also avoid this difficulty.

The third and the last question is on equilibrium outcomes in a quantity-setting duopoly. It can be shown that each joint profit maximizing quantity pair is attained as the unique final outcome of a CNPE: the proof is similar to that in the price-setting duopoly. But it is not certain whether or not there exist any other equilibrium outcomes.

References

Bhaskar, V. (1989), Quick Responses in Duopoly Ensure Monopoly Pricing, *Economics Letters* 29, 103-107.

Chwe, M.S.-K. (1992), Farsighted Coalitional Stability, mimeo, Department of Economics, University of Chicago.

Greenberg, J. (1990), *The Theory of Social Situations: An Alternative Game Theoretic Approach*, Cambridge University Press.

Harsanyi, J.C. (1974), An Equilibrium-Point Interpretation of Stable Sets and a Proposed Alternative Definition, *Management Science* 20, 1472-1495.

Kalai, E. (1981), Preplay Negotiations and the Prisoner's Dilemma, *Mathematical Social Sciences* 1, 375-379.

Maskin, E. and J. Tirole (1988), A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles, *Econometrica* 56, 571-599.

Muto, S. and D. Okada (1992), Von Neumann-Morgenstern Stable Sets in a Price-Setting Duopoly, TERG discussion paper No. 103, Faculty of Economics, Tohoku University, Japan.

Von Neumann, J. and O. Morgenstern (1953), *The Theory of Games and Economic Behavior* (3rd ed.), Princeton University Press.

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